TRINITY COLLEGE

# Year 11 Mathematics Specialist 1,2
# Test 4 2021

## Section 1 Calculator Free
**Trigonometry**

**STUDENT'S NAME** ______________________________

**DATE**: Friday 30 July **TIME:** 33 minutes **MARKS**: 33

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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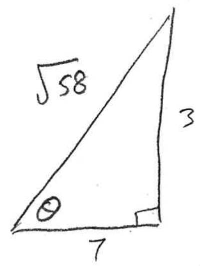
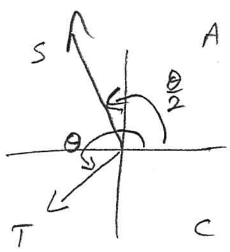
1. (5 marks)

Given $\tan\vartheta = \frac{3}{7}$ and $180° \le \vartheta \le 270°$, determine

(a) the exact value of $\sin\vartheta$ [2]

$-\frac{3}{\sqrt{58}}$




(b) the exact value of $\cos\frac{\vartheta}{2}$ [3]

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\pm\sqrt{\frac{\cos\theta + 1}{2}} = \cos\frac{\theta}{2}$

$-\sqrt{\frac{\frac{-7}{\sqrt{58}} + 1}{2}} = \cos\frac{\theta}{2}$ (QUAD 2)

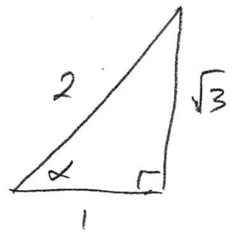
2. (10 marks)

(a) Express $\sqrt{3}\cos x + \sin x$ in the form $R\sin(x+\alpha)$ for $\alpha$, an acute angle in radians. [4]

$$2\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$$

$$= 2\sin(x+\alpha)$$

$$= 2\sin\left(x + \frac{\pi}{3}\right)$$




$\alpha = \frac{\pi}{3}$

(b) Determine the minimum value of the expression in (a) and determine the smallest positive value of $x$ for which this occurs. [3]

MIN $= -2$ WHEN $x + \frac{\pi}{3} = \frac{3\pi}{2}$

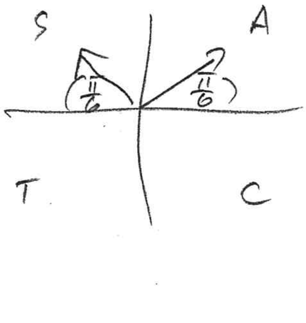
$x = \frac{7\pi}{6}$

(c) Hence or otherwise, solve the equation $\sqrt{3}\cos x + \sin x = 1$ for $0 \le x \le 2\pi$. [4]

$$2\sin\left(x+\frac{\pi}{3}\right) = 1$$

$$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$$

REF ANGLE $= \frac{\pi}{6}$




$x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$

$x = \cancel{-\frac{\pi}{6}}, \frac{\pi}{2}, \frac{11\pi}{6}$

$= \frac{\pi}{2}, \frac{11\pi}{6}$

3. (8 marks)

[4]

(a) Prove $\dfrac{1-\cos 2\vartheta+\sin 2\vartheta}{1+\cos 2\vartheta+\sin 2\vartheta}=\tan\vartheta$

$$LHS = \frac{1-(1-2\sin^2\theta)+\sin 2\theta}{1+(2\cos^2\theta-1)+\sin 2\theta}$$

$$= \frac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\,\cancel{(\sin\theta+\cos\theta)}}{2\cos\theta\,\cancel{(\cos\theta+\sin\theta)}}$$

$$= \tan\theta$$

$$= RHS$$

[4]

(b) Using the result of (a), show $\tan 15^\circ = 2-\sqrt{3}$

$$\tan 15^\circ = \frac{1-\cos 30^\circ+\sin 30^\circ}{1+\cos 30^\circ+\sin 30^\circ}$$

$$= \frac{1-\frac{\sqrt{3}}{2}+\frac{1}{2}}{1+\frac{\sqrt{3}}{2}+\frac{1}{2}}$$

$$= \frac{\frac{3}{2}-\frac{\sqrt{3}}{2}}{\frac{3}{2}+\frac{\sqrt{3}}{2}}$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{9-2\sqrt{3}+3}{9-3}$$

$$= \frac{12-2\sqrt{3}}{6}$$

$$= 2-\sqrt{3}$$

4. (5 marks)

If $(\sin A+\cos B)^2+(\cos A+\sin B)^2=3$, determine two possible values for the angle $(A+B)$ where $0\leq(A+B)\leq 2\pi$.

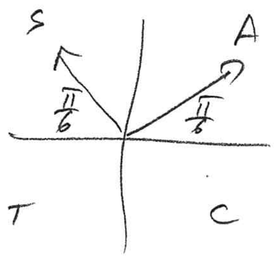
$$\sin^2 A + 2\sin A\cos B + \cos B^2 + \cos A^2 + 2\sin B\cos A + \sin^2 B = 3$$

$$1 + 2\sin A\cos B + 2\sin B\cos A + 1 = 3$$

$$2(\sin A\cos B + \cos A\sin B) = 1$$

$$\sin(A+B) = \frac{1}{2}$$

$$A+B = \frac{\pi}{6}, \frac{5\pi}{6}$$




REF ANGLE $= \frac{\pi}{6}$

5. (6 marks)

Solve $0.5\sec(2\vartheta-\frac{\pi}{3})=1$

$$\sec\left(2\theta-\frac{\pi}{3}\right) = 2$$

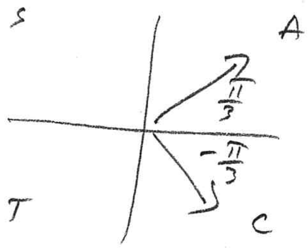
$$\cos\left(2\theta-\frac{\pi}{3}\right) = \frac{1}{2}$$

REF ANGLE $= \frac{\pi}{3}$

$$2\theta-\frac{\pi}{3} = \frac{\pi}{3} \qquad 2\theta-\frac{\pi}{3} = -\frac{\pi}{3}$$

$$2\theta = \frac{2\pi}{3} + 2\pi n \qquad 2\theta = 0 + 2\pi n$$

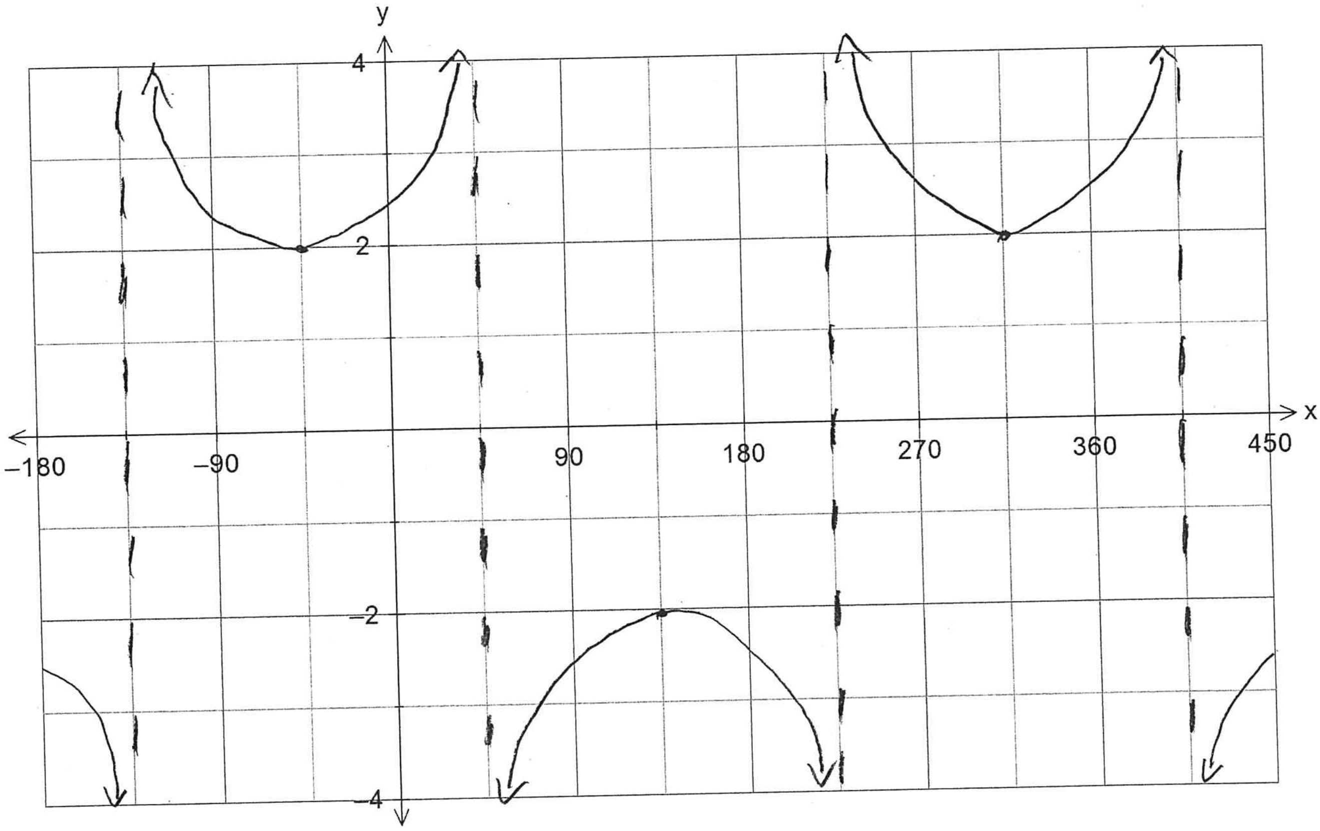
$$\theta = \frac{\pi}{3} + n\pi \qquad \theta = \pi n$$




$$n \in \mathbb{Z}$$

6. (4 marks)

Sketch the graph of $y = -2\text{cosec}(x - 45°)$ on the axes below.



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# Year 11 Mathematics Specialist 1,2
# Test 4 2021

Section 2 Calculator Assumed

**Trigonometry**

**STUDENT'S NAME** SOLNS.

**DATE:** Friday 30 July **TIME:** 17 minutes **MARKS:** 17

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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7. (5 marks)

Prove $\dfrac{1+\cot\alpha}{\operatorname{cosec}\alpha} - \dfrac{\sec}{\cot\alpha + \tan\alpha} = \cos\alpha$

$$LHS = \frac{1+\frac{\cos\alpha}{\sin\alpha}}{\frac{1}{\sin\alpha}} - \frac{\frac{1}{\cos\alpha}}{\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{\cos\alpha}}$$

$$= \frac{\frac{\sin\alpha}{\sin\alpha}+\frac{\cos\alpha}{\sin\alpha}}{\frac{1}{\sin\alpha}} - \frac{\frac{1}{\cos\alpha}}{\frac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha\cos\alpha}}$$

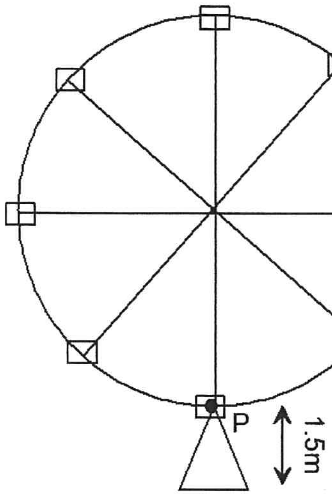
$$= \frac{\sin\alpha+\cos\alpha}{\cancel{\sin\alpha}} \times \frac{\cancel{\sin\alpha}}{1} - \frac{1}{\cancel{\cos\alpha}} \times \frac{\sin\alpha\cancel{\cos\alpha}}{1}$$

$$= \sin\alpha + \cos\alpha - \sin\alpha$$

$$= \cos\alpha$$

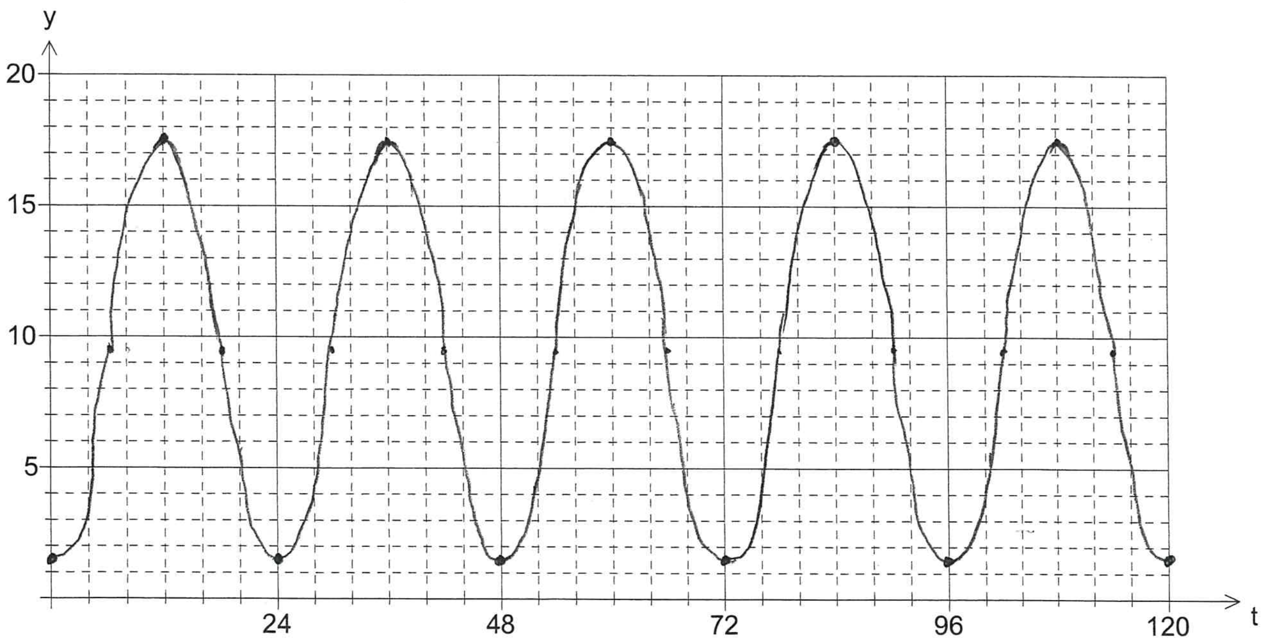
8. (12 marks)

The height above ground of a person sitting in a cart on a Ferris Wheel can be modelled by a trigonometric function.




(a) Paulo is sitting on a chair in a Ferris Wheel of radius 8m. His starting position is 1.5 m above ground as shown in the diagram above. The Ferris Wheel moves around anticlockwise, at a constant velocity, one revolution every 24 seconds.

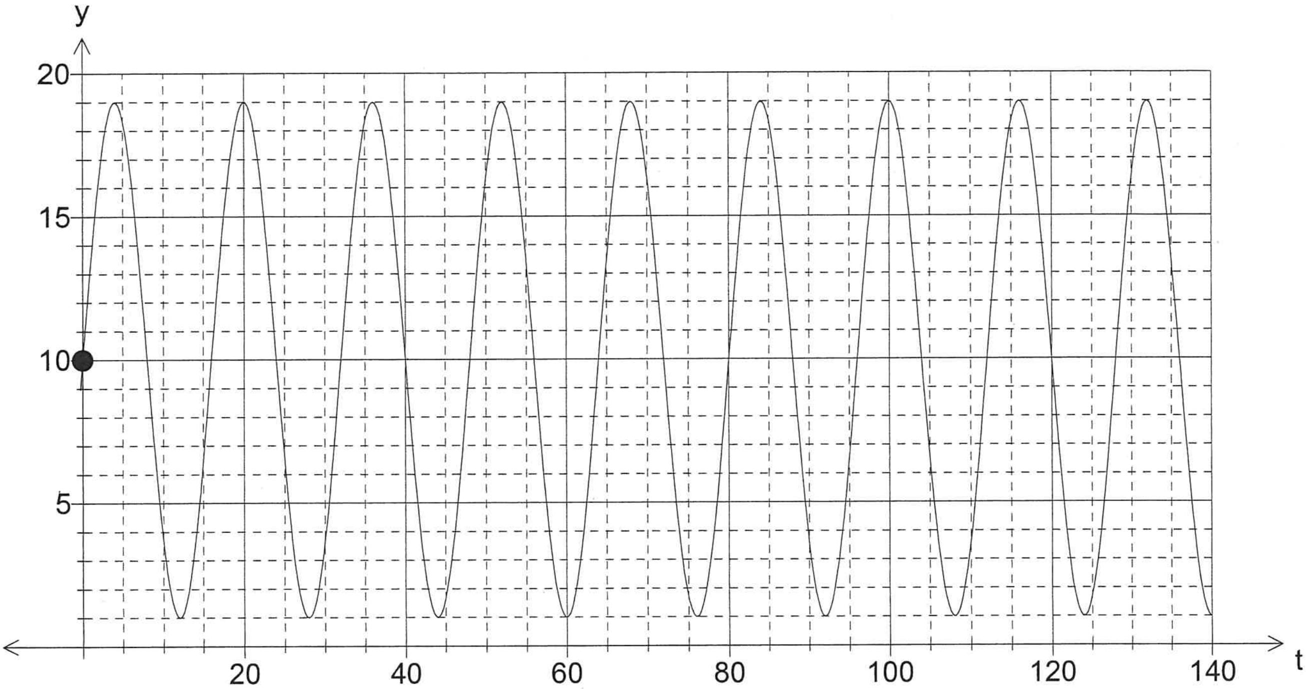
(i) Draw a graph representing how Paulo's height changes over time each revolution, given t is in seconds and y is Paulo's height in metres. [3]




(ii) Determine the maximum height reached by Paulo and the times this occurs if the Ferris Wheel stops after 2 minutes. [3]

17.5 m 12, 36, 60, 84, 108 sec

(b) The graph below shows Matia's height above ground over time when sitting in a cart of a different Ferris wheel to Paulo. This Ferris Wheel also moves around anticlockwise.




Matia's height above ground over time is modelled by the equation

$y = a \sin bt + c$

where y is Matia's height above the ground in m, at time t, secs and a, b and c are constants.

(i) What is the radius of this Ferris Wheel? 9m [1]

(ii) Determine the time taken for one revolution. [1]

16 sec

(iii) Determine the value of a, b and c. [3]

a 9m

b 22.5

c 10m

(iv) On the Ferris Wheel below, indicate Matia's starting position. [1]